

## ADDENDUM TO THE PAPER

### SURFACE SINGULARITY AND CRACK PROPAGATION†

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The purpose of this Addendum is to provide new experimental results which corroborate the findings of the paper.

As has been theoretically shown in this paper, the front edge of a planar Mode I crack that propagates in an elastic isotropic plate in a plane normal to the plate surfaces must terminate at the surfaces with a certain angle  $\beta$  depending only on Poisson ratio  $\nu$ . As far as comparisons with experimental results are concerned, some limited evidence was shown in Figs. 12 and 13 of the paper. The agreement was essentially qualitative, confirming that for  $\nu > 0$  angle  $\beta$  exceeds  $90^\circ$  such that the terminal point of crack edge trails in propagation behind the adjacent interior points of crack edge. However, the observed angle in these tests was about  $10^\circ$  higher than the obtained theoretical value  $\beta = 102^\circ$  for  $\nu = 0.33$ . The discrepancy may have been due to inelastic behavior near the crack edge, particularly due to the fact that these were fatigue tests which were no doubt influenced by cyclic plastic deformation near the edge.

After publication of the paper, more relevant test results were provided to the writers by Professor Stanley T. Rolfe from the University of Kansas at Lawrence. These tests, the photographs of which are exhibited in Figs. 15(a)–(c), with the permission of Professor Rolfe,



(a)

†Z. P. Bažant and L. F. Estenssoro, Surface singularity and crack propagation. *Int. J. Solids Structures* 15, 405–426 (1978).



(b)



(c)

Fig. 15 (a, b, c). Three fracture tests of S. T. Rolfe compared with theoretical direction of the crack at the surface points (reproduced with the permission of S. T. Rolfe, University of Kansas at Lawrence).

were carried out on structural steel in static loading, tested according to the E-399 ASTM Standard. For further comments on this type of test the reader may consult pages 351-353 of Ref.[26].

The theoretical lines of inclination  $\beta = 101^\circ$  with the plate surface have been drawn in Fig. 15. It is seen that they very well agree with the observed terminal directions of the first crack arrest marks, which correspond to an essentially elastic fracture stage. In fact, the theoretical lines do not appear to deviate at all from these observed directions within the resolution of the photographs.

This comparison presents an excellent experimental confirmation of the theory presented in the paper.

#### REFERENCE

26. S. T. Rolfe and J. M. Barsom, *Fracture and Fatigue Control in Structures*. Prentice Hall, Englewood Cliffs, New Jersey (1977).